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Optimization of site investigation program for reliability assessment of undrained slope using Spearman rank correlation coefficient

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1. Abstract

Site investigation programs (e.g., boreholes) are crucial in characterizing soil properties and stratigraphic configurations. However, the traditional borehole patterns are generally of equally spaced distribution for the slope design, and the locations and total number of boreholes are considerably determined depending on engineers' experience, which may lead to cost-inefficient geotechnical design, especially considering the soil spatial variability. To address this dilemma, this paper presents a Spearman rank correlation coefficient-based scheme to optimize site investigation in slope design, where both locations and total number of boreholes are optimized. Conditional random field simulations are performed to consider the effect of the borehole data on the estimation of the soil property distribution. The superiority of the proposed method to the traditional method is illustrated by a comparison study in an undrained slope example. In this example, the accuracy of the characteristics of the slope (i.e., the factor of safety, location of slip surface, and sliding volume), robustness of the estimated characteristics of the slope, and risk reduction are examined. The comparison results show the effectiveness of the proposed method in accurately estimating the characteristics of the slope without prior knowledge about the slip surface, since the slip surface is unknown for most practical cases prior to the site investigation. The most robust estimate results and risk reduction are obtained using the proposed method. This study can also provide useful references to build an adaptive unequally spaced borehole pattern in practice.

2. Background and objectives

- Traditional borehole patterns are generally equally spaced for site investigations. Previous studies show that the optimal borehole patterns are generally related to the failure mechanism of geotechnical systems (e.g., the slope). The soil elements at the influence zones that control the failure mechanism of geotechnical systems are more critical for the geotechnical problems, indicating that traditional equally spaced borehole patterns are more likely cost-inefficient.
- This study aims to propose an effective approach to build an adaptive unequally spaced borehole pattern considering the spatial soil variability based on correlation analysis for the undrained slope stability analysis.

3. Methodology and implementation procedures

The soil spatial variability is characterized by the random field theory. The effect of the boreholes is captured by the kriging-based conditional random field simulations. The Spearman rank correlation coefficient (ρ_{Spearman}) between each soil element and factor of safety (FS) can be calculated by Eq. (1). The effectiveness of the additional borehole along all potential horizontal locations is evaluated based on the mean Spearman rank correlation coefficient of the soil elements from the borehole. The characteristics of the slope can be derived from the Monte-Carlo simulations (MCS). Flowchart for the proposed method is plotted in Figure 1. One unconditional random field simulation with soil properties in Table 1 is selected as the "true" slope for illustration as shown in Figure 2. The FS and ρ_{Spearman} are converged with 1000 MCS in Figure 3. Therefore, 1000 runs of MCS are performed in this study.

$$\rho_{Spearman}(\boldsymbol{s}_j,\boldsymbol{FS})=\frac{\sum_{i=1}^{N_p}(n_{1i}-\bar{n}_1)(n_{2i}-\bar{n}_2)}{\sqrt{\sum_{i=1}^{N_p}(n_{1i}-\bar{n}_1)^2}\sqrt{\sum_{i=1}^{N_p}(n_{2i}-\bar{n}_2)^2}} \quad (1)$$

where n_{1i} and n_{2i} are the ascending or descending sorted positions determined by the values of each element in j^{th} soil element s_j and FS with N_p random field simulations, respectively; $\bar{n}_1$ is the mean of n_{1i} (i=1, 2, …, N_p); $\bar{n}_2$ is the mean of n_{2i} (i=1, 2, …, N_p).

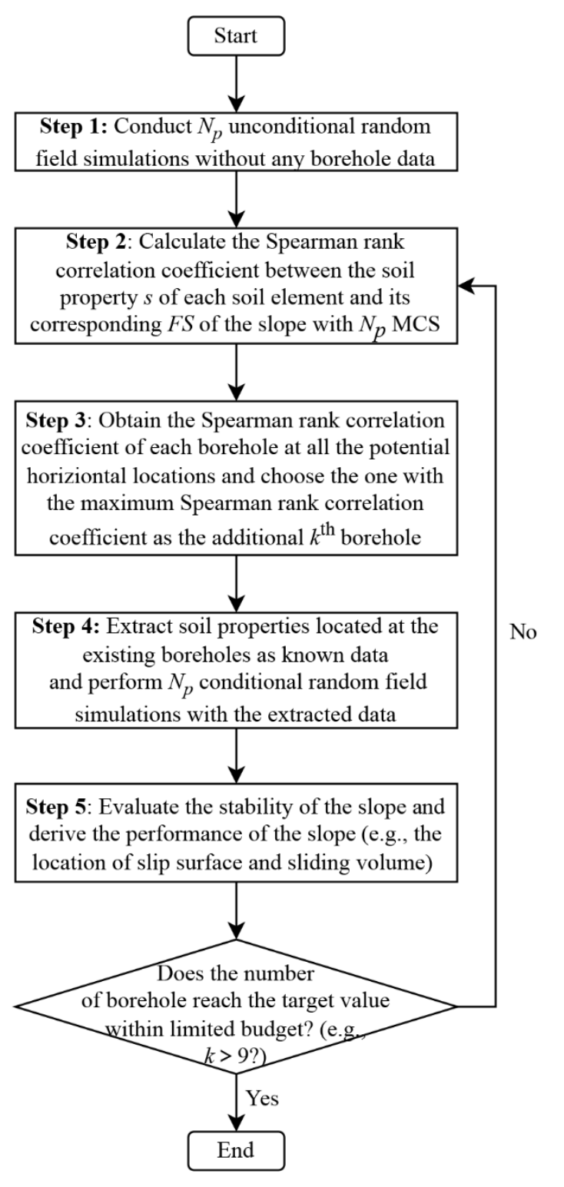

Start
Step 1: Conduct N_p unconditional random field simulations without any borehole data
Step 2: Calculate the Spearman rank correlation coefficient between the soil property s of each soil element and its corresponding FS of the slope with N_p MCS
Step 3: Obtain the Spearman rank correlation coefficient of each borehole at all the potential horizontal locations and choose the one with the maximum Spearman rank correlation coefficient as the additional k^{th} borehole
Step 4: Extract soil properties located at the existing boreholes as known data and perform N_p conditional random field simulations with the extracted data
Step 5: Evaluate the stability of the slope and derive the performance of the slope (e.g., the location of slip surface and sliding volume)
Does the number of borehole reach the target value within limited budget? (e.g., $k > 9$?)
No
Yes
End

Figure 1. Flow chart of the proposed method

Table 1. Statistics of the soil properties for the example slope problem

Parameter	Value
Density, ρ (kg/m³)	2000
Young's modulus, E (MPa)	100
Poisson's ratio, v	0.30
Mean of undrained shear strength, c_u (kPa)	40
COV of undrained shear strength c_u	0.3
Horizontal scale of fluctuation, λ_{lnx} (m)	40
Vertical scale of fluctuation, λ_{lny} (m)	4

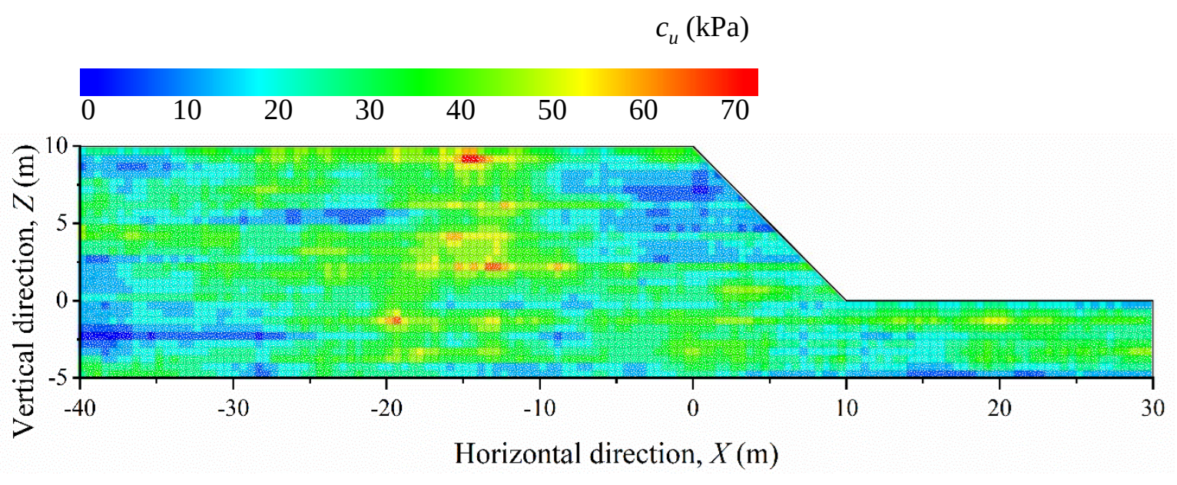
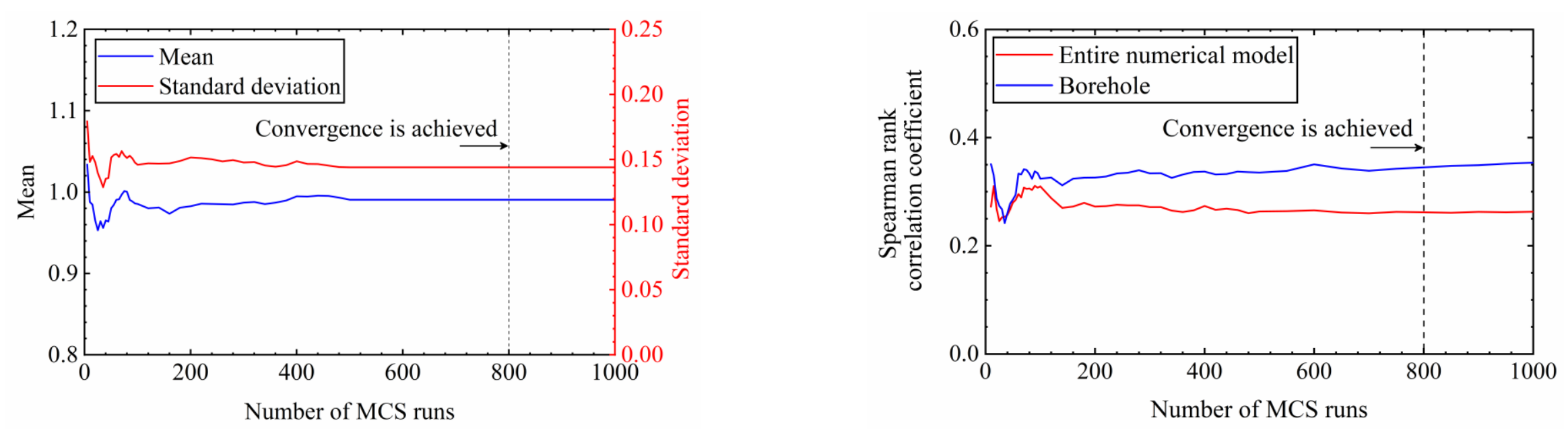


Figure 2. Spatially variable soils in the "true" slope



(a) Convergence of FS (b) Convergence of Spearman correlation coefficient

Figure 3. Convergence of the FS and Spearman correlation coefficient with the number of MCS runs

4. Example application for the slope problem

The stability of the slope in terms of the FS is obtained by the strength reduction method built in the FLAC3D version 7.0 while the location of the slip surface can be determined by the nodal displacement of the numerical elements, taking the slip surface derive from the maximum shear strain increment (SSI) method as a benchmark as shown in Figure 4. The uncertainty of the location of the slip surface can be characterized by the three controlling points in Figure 5. After the location of the slip surface is determined, the sliding volume can estimated by the number of the sliding soil elements.

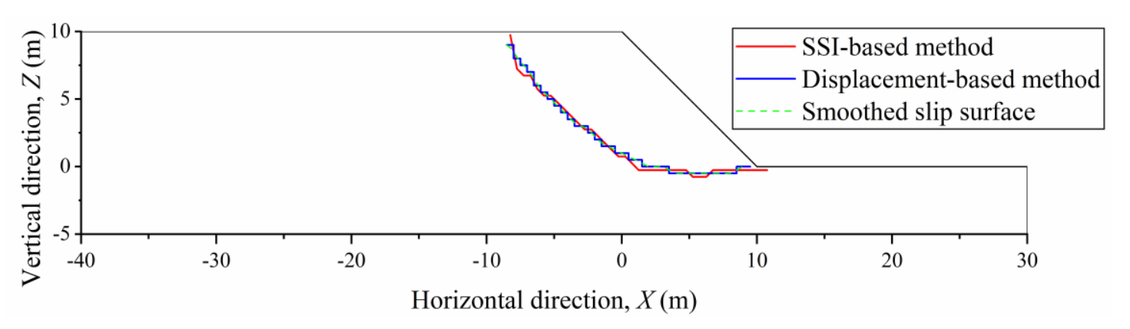


Figure 4. Location of the slip surface, which is determined by 35% of the maximum nodal displacement and maximum shear strain increment

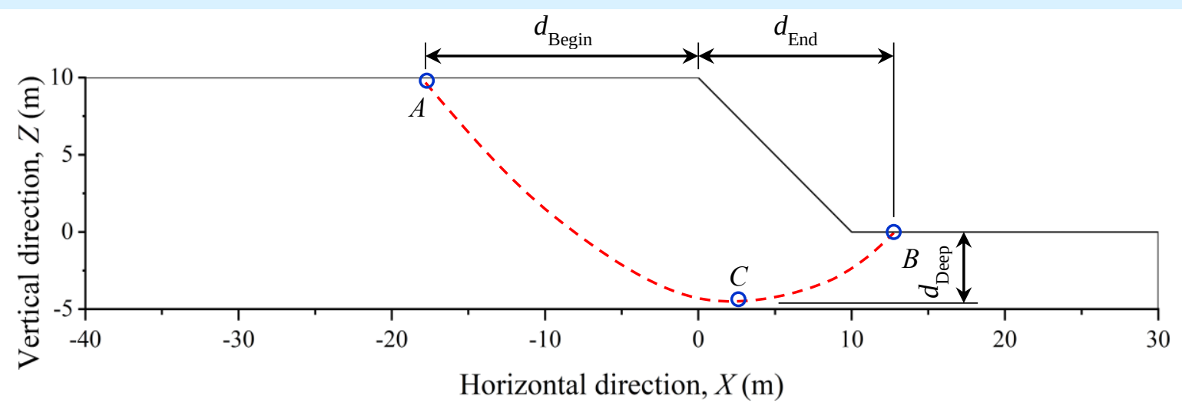
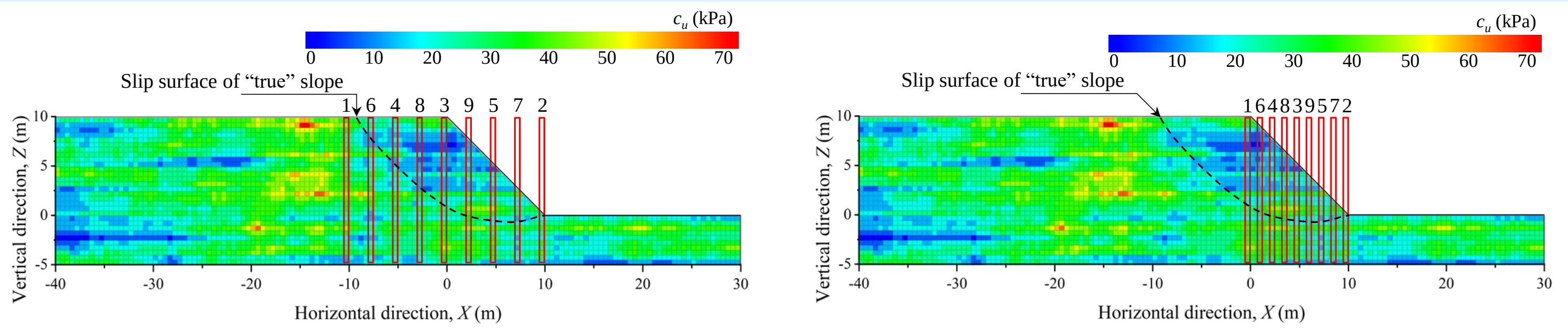
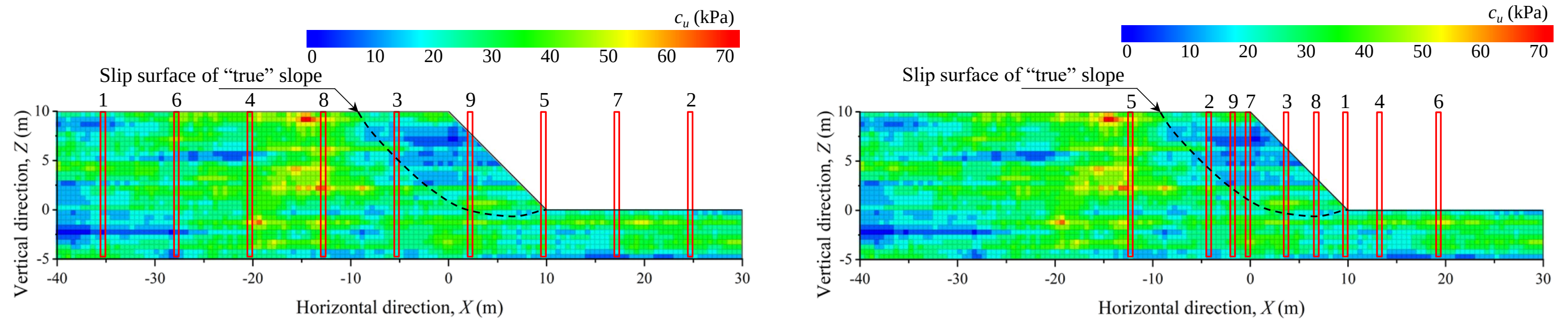


Figure 5. Location of the slip surface characterized using three controlling points

The influence zone is estimated depending on the engineer's experience in the traditional method. Three borehole patterns from the traditional method and the borehole pattern from the proposed method are adopted for the comparison study as shown in Figure 6.



(a) Traditional method with good judgement (b) Traditional method with moderate judgement



(c) Traditional method with poor judgement (d) Proposed method

Figure 6. Borehole patterns from the traditional method and proposed method for the comparison study

The FS can be well estimated for all the borehole patterns when more than three boreholes are applied as shown in Figure 7. The borehole patterns from the proposed method and traditional method with good judgement can accurately estimate the location of the slip surface and sliding volume while remaining two borehole patterns will cause large errors when sufficient boreholes are adopted as shown in Figure 8.

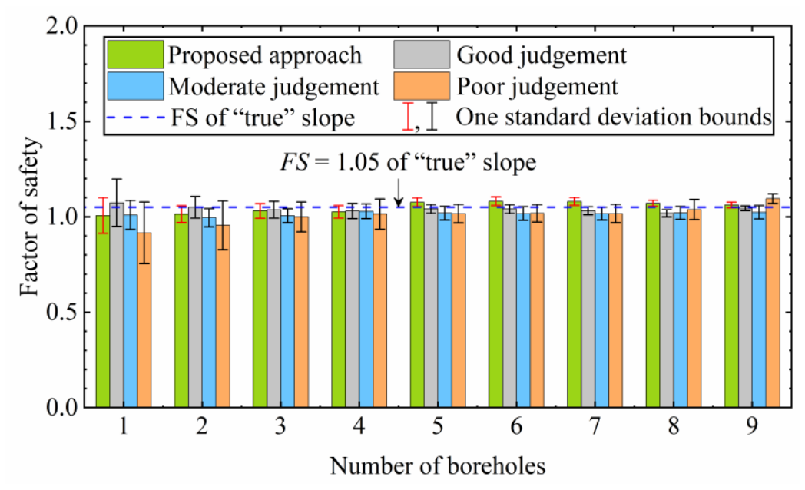


Figure 7. A comparison study of the estimated FS among different borehole patterns

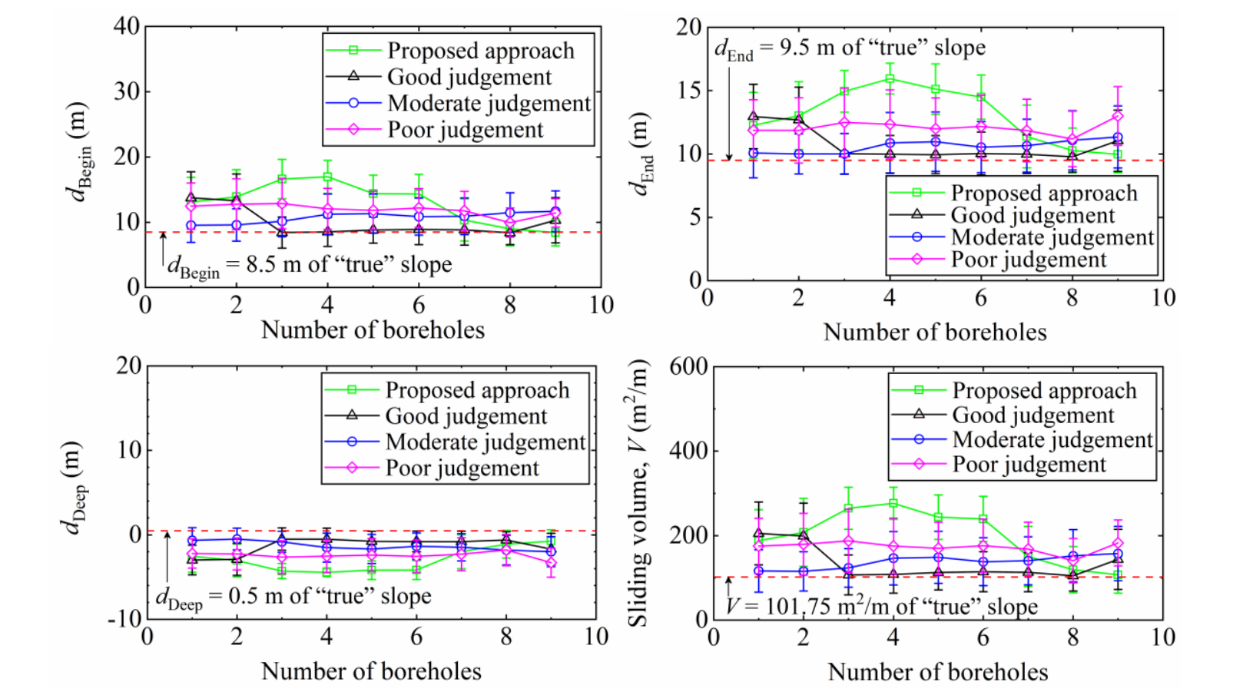
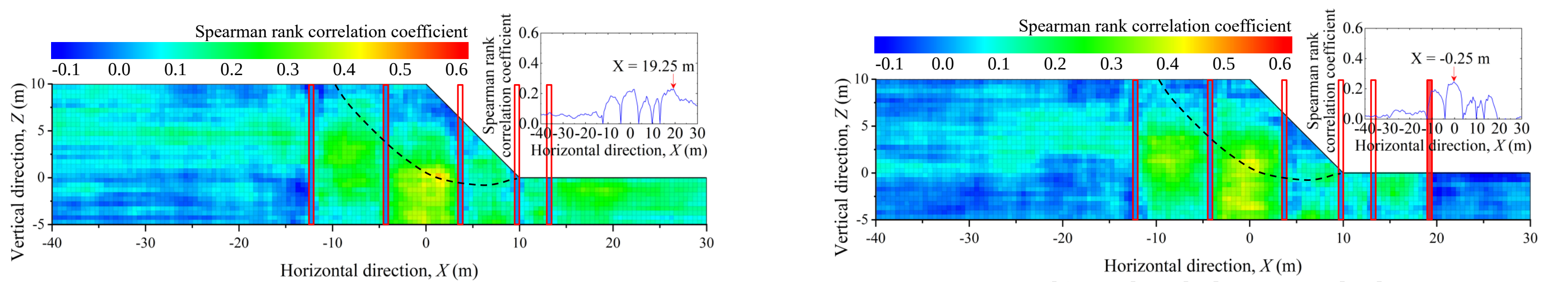


Figure 8. A comparison study of estimate of the characteristics of the slope

Although the fourth, fifth and sixth boreholes are not located at the influence zone, the sliding area is well bracketed by the fourth and fifth boreholes from the proposed method, since a small error will be obtained for the estimation. The sixth borehole can be considered as a part of the effort to automatically identify the influence zone as shown in Figure 9.



(a) Five boreholes are applied (b) Six boreholes are applied

Figure 9. Illustration of the effect of the sixth borehole from the proposed method

Figure 10 shows that the final influence zone is derived as the area between the fifth and sixth boreholes, while the remaining area is a low-correlated soil zone. Therefore, it is effective to automatically estimate the influence zone for the slope with the proposed method, even if the initial investigation area is considerably conservatively selected.

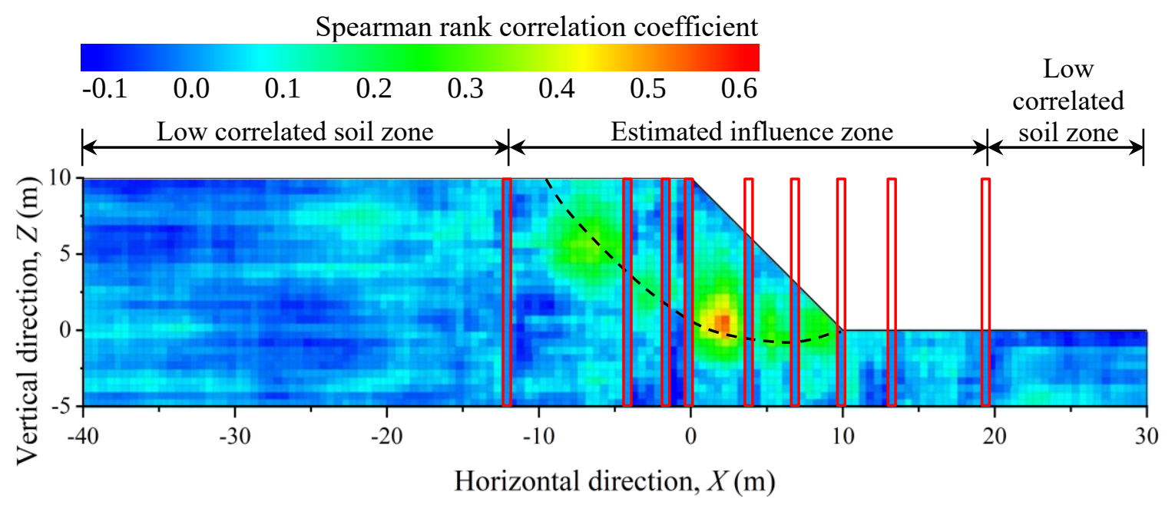


Figure 10. Estimated influence zone by the proposed method

5. Robustness analysis and risk assessment

In addition to the characteristics of the slope, uncertainty reduction and risk reduction are aspects of slope design. The uncertainty reduction can be characterized by a weighted averaged signal-to-noise ratio (SNR) by Eq. (5). Figure 11 shows that most robust estimate of the characteristic of the slope can be obtained by the proposed method.

The risk for the slope can be assessed by the expected total cost of the site investigation effort. The total cost C_{total} with different numbers of boreholes (N_{BH}) can be calculated by the Eq. (6). The average cost of each borehole is assumed to be C_{BH} = \$AUD 5,000, and the loss of making the false unsafe assessment of the slope stability is assumed to be C_{false} = \$AUD 150,000. A minimum expected total cost can be obtained by the proposed method as shown in Figure 12.

$$SNR_{FS}=10\log_{10}\left(\frac{\mu_{FS}^2}{\sigma_{FS}^2}\right) \quad (2)$$

$$SNR_L=\frac{1}{3}(SNR_A+SNR_B+SNR_C) \quad (3a)$$

$$SNR_A=10\log_{10}\left(\frac{\mu_{d_{\text{Begin}}}^2}{\sigma_{d_{\text{Begin}}}^2}\right) \quad (3b)$$

$$SNR_B=10\log_{10}\left(\frac{\mu_{d_{\text{End}}}^2}{\sigma_{d_{\text{End}}}^2}\right) \quad (3c)$$

$$SNR_C=10\log_{10}\left(\frac{\mu_{d_{\text{Deep}}}^2}{\sigma_{d_{\text{Deep}}}^2}\right) \quad (3d)$$

$$SNR_V=10\log_{10}\left(\frac{\mu_V^2}{\sigma_V^2}\right) \quad (4)$$

$$S=w_{FS}SNR_{FS}+w_L SNR_L+w_V SNR_V \quad (5)$$

$$C_{\text{total}}=N_{\text{BH}}\cdot C_{\text{BH}}+P_f\cdot C_{\text{false}} \quad (6)$$

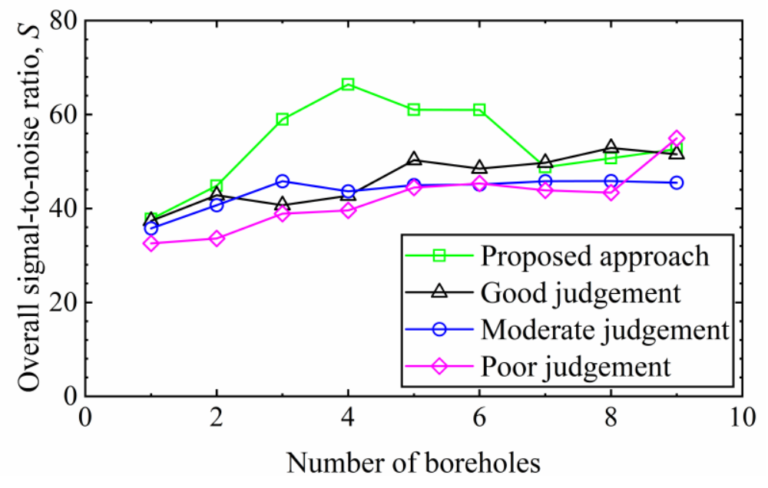


Figure 11. Overall robustness with the increase in number of boreholes

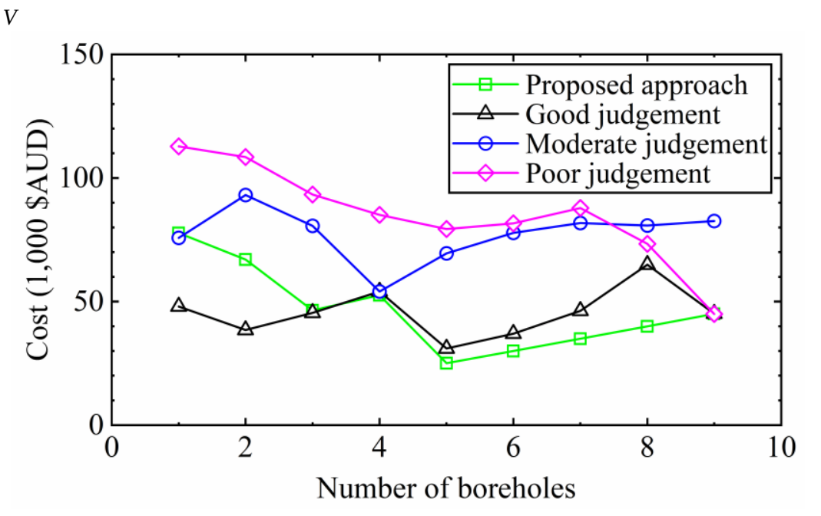


Figure 12. Risk assessment with the increase in number of boreholes

6. Conclusions

The proposed method can accurately estimate the characteristics of the slope (e.g., FS, location of slip surface and sliding volume). Most uncertainty reduction and robust estimation of the characteristics can be reached by the proposed method. The proposed method provides a reference to build an adaptive unequally spaced borehole pattern without prior knowledge about the slip surface in practice.

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